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MATHEMATICAL ENGINEERING, A FIVE YEAR PROGRAM.
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THIS REPORT OF THE COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS (CUPM) DESCRIBES A PROPOSED FIVE-YEAR PROGRAM FOR THE UNDERGRADUATE PREPARATION OF STUDENTS IN MATHEMATICAL ENGINEERING. THE MAJOR PURPOSES OF THIS PROGRAM ARE TO PREPARE STUDENTS IN THE MATHEMATICAL SCIENCES, TO ENABLE THEM TO APPLY THIS EDUCATION TO THE COMPLICATED ENGINEERING PROBLEMS OF THE SPACE EFFORT, AND THEREBY, TO PROVIDE THE NATION WITH TECHNICAL MANPOWER HAVING A WIDE BACKGROUND IN BOTH MATHEMATICS AND ENGINEERING. INTEREST IN THIS KIND OF PROGRAM BUT WITH A DIFFERENT SPECIALIZATION ALSO HAS BEEN SHOWN BY SUCH INDUSTRIES AS ELECTRONICS AND COMMUNICATION. SINCE THE MATHEMATICAL ENGINEERING PROGRAM INVOLVES A HEAVY CONCENTRATION IN MATHEMATICS AS WELL AS INSTRUCTION IN ENGINEERING INVOLVING BOTH APPLICATION AND THEORY, THE COMMITTEE DEVELOPED THE NOTION OF A COMMON CORE OF MATERIAL IN THE BASIC PHYSICAL SCIENCES AND, MORE EXTENSIVELY, IN THE MATHEMATICAL SCIENCES. THE CORE, IN TURN, IS COMPLEMENTED BY A NUMBER OF OPTIONS, WHICH ARE MORE SPECIALIZED DEVELOPMENTS IN DEPTH AND ENSURE THAT THE STUDENT WILL BE INTRODUCED TO AT LEAST ONE BRANCH OF ENGINEERING. ORBIT MECHANICS, OPERATIONS RESEARCH, AND CONTROL THEORY ARE THREE OPTIONS WHICH ARE DESCRIBED IN DETAIL IN THIS REPORT. THIS DOCUMENT IS ALSO AVAILABLE WITHOUT CHARGE FROM CUPM CENTRAL OFFICE, P. O. BOX 1024, BERKELEY, CALIFORNIA 94701. (RP)

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MATHEMATICAL ASSOCIATION OF AMERICA

MATHEMATICAL ENGINEERING A FIVE YEAR PROGRAM

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

OCTOBER 1966

Panel on Mathematics for the Physical Sciences and Engineering

MATHEMATICAL ENGINEERING

A FIVE YEAR PROGRAM

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS
Mathematical Association of America
October, 1966

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

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COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS
October 1966

PANEL ON MATHEMATICS FOR THE PHYSICAL SCIENCES AND ENGINEERING

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The CUPM Panel on Mathematics for the Physical Sciences and Engineering presents this report to Departments of Mathematics and Engineering as one possible means of alleviating the present drastic shortage of applied mathematicians. It is a suggestion, rather than a recommendation.

The program presented here would never have been conceived or designed without suggestions and advice from a large number of people, with interests spread widely over mathematics and its applications. To all these advisors we extend our sincere thanks for their invaluable assistance. Most particularly we wish to thank those who went to the trouble of supplying us with detailed outlines of one or more courses: R. E. Barlow, Z. W. Birnbaum, E. H. Brock, F. E. C. Culik, A. Deprit, A. S. Galbraith, P. Herget, M. R. Hestenes, M. W. Johnson, Jr., W. L. Maxwell, H. Pollard, J. F. Price, F. Proschan, J. W. Siry, F. W. Sinden, D. W. Walkup, R. H. Wilson, Jr., J. Wolfowitz.

INTRODUCTION

During a conversation at the mathematics meetings in January, 1964, the late S. S. Wilks of Princeton University pointed out the absence of any special large-scale efforts to provide technical personnel for the nation's space program. As he saw it, there was a continuing need for persons really well trained in the mathematical sciences and able to apply these to the complicated engineering problems of the space effort. His aim was to provide technical manpower specifically prepared for a strong combination of mathematics and engineering—in addition to the more customary converts from a great variety of backgrounds. He felt that CUPM, with its tradition of pervasive concern for all aspects of collegiate mathematical education, might undertake a study of this problem. The present report is, in part, a response to his ideas.

A pair of meetings with representatives of NASA and the space industry confirmed the eminent need for the projected product. At the same time it became clear that many other industries, such as electronics and communication, would have an equal interest in a mathematical engineer with the same basic background but possibly somewhat different specialization. All these industries face innumerable engineering problems with a common need for extensive and sophisticated mathematical analysis. For the solution of such problems it is no longer true that the prime requirement is a good physical intuition—rather, one also needs a well developed mathematical intuition. Thus the mathematical engineering program must involve a heavy concentration in mathematics, but with a choice of topics that will give a useful basis for applications as well as solid grounding in theory. As the Panel came to grips with this multiplicity of purposes, it developed the notion of a common core, for all mathematical engineers, of material in the basic physical sciences and, more extensively, in the mathematical sciences. The core, in turn, is complemented by a number of options, which are more specialized developments in depth and assure that the student will be fairly well acquainted with at least one branch of engineering. Orbit mechanics, operations research, and control theory are three options which are developed in detail in the present report.

It turns out that a minimum of five years, rather than four, is needed to carry out the desired sequences in depth. It is not immediately obvious what the student's area of concentration should be called. The dual emphasis on mathematics and engineering makes either field conceivable, and in fact, the program comes close to being recognizable as a master's program in applied mathematics. However, the heavy emphasis on the physical sciences, the concern in each option with the building of mathematical models, and the rather heavily prescriptive nature of the program make a realization within the engineering school more suitable. Wherever

the program may appear within an institution's offerings, it should involve close cooperation between the mathematicians and the engineers.

A natural question also arises as to the possible fields of further graduate study which a student could enter on completion of the present program. It is our opinion that relatively little, if any, "remedial" work will be necessary to qualify the student for a doctoral program in applied mathematics or, depending on the particular option, in engineering science, or in industrial or electrical engineering. Whether or not a master's degree should be given for the completion of this program is a matter for the offering institution to decide. As remarked above, the content of the program is of the right order of magnitude for this degree. Other considerations (e.g., requirement of a thesis) may be deciding factors.

A number of additional remarks about the program are in order. A most important aspect is the flexibility which would automatically be built into the mathematical engineer. With such a background, and with a considerable facility in making connections between the real world and mathematical models thereof, such a man could easily retrain himself, say, from space science to oceanography, if a sudden shift of present national interest should make this desirable. Secondly, the similarity in spirit of this program to the recently introduced engineering physics and engineering science programs is worth noting. The idea of these programs was to give the student a solid background of the kind of physics that would be useful in a wide variety of engineering applications, along with enough engineering subjects to impart some feeling for the kinds of problems he would encounter. There is now no question of the value of such training. In just the same way, mathematical engineering combines a solid foundation in major areas of applicable mathematics with real strength in some particular area of engineering, and experience in connecting the two. Incidentally, it should be remarked that mathematical engineering has existed for some years, much in the spirit of the present report, at several universities in the Netherlands. It seems to be a successful program from the point of view of both employment opportunity and preparation for further graduate work.

DESCRIPTION OF PROGRAM

As remarked above, the program is constructed around a core consisting of a heavy concentration of mathematics and the physical sciences. Attached to the core there may be many options, each providing motivation, application, and extension of the core material to some phase of engineering. The core is fairly well defined and will probably not vary greatly from one institution to another. The options, on the other hand, will necessarily have much local flavor both in their general subject matter and in the particular courses that compose them. The three options that we present here are thus to be regarded as samples of what can be done.

THE CORE

Modern engineering is built upon a three-part foundation consisting of mathematics, the physical sciences, and automatic computing. The last of these is a newcomer, and its precise role and manner of development are still matters of speculation, but there is no question as to its basic importance. Those three topics, then, compose the core.

The mathematical portion, which is, for this program, the most extensive, is based on the CUPM <u>Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists</u> (published in 1962, reissued in 1965, to be revised in 1967). The courses recommended there as preparation for graduate work have been modified somewhat and about nine semester hours have been added, much of it as additional work in topics already begun. This gives us the following list (the initial number refers to the course outlines given in the Appendix, and "hours" means "semester hours"):

Calculus and Linear Algebra (12-15 hours)
1. Functions of Several Variables (3 hours)
2. Intermediate Ordinary Differential Equations (3 hours)
4. Numerical Analysis (3 hours)
5. Probability and Statistics (6 hours)
6. Complex Variables (3 hours)
7. Functional Analysis (3 hours)
10. Partial Differential Equations (6 hours)
11. Optimization (3 hours)

Discussion of the individual courses is deferred until the whole core has been described.

The resulting 42-45 hours of mathematics in the core is about the magnitude of a good undergraduate major in mathematics, but the emphasis

is quite different. This program stresses analysis heavily. Indeed, the minimal treatment of algebra and geometry is perhaps the most vulnerable point of this curriculum. However, for the foreseeable future the topics included are certainly of first importance. Other mathematical topics neede in certain courses can be developed when needed to the extent required.

We recognize full well that the value of these courses depends on the spirit in which they are taught. One must keep in mind that their ultimate purpose is application, either directly or as preparation for more obviously applicable topics. Such courses as Partial Differential Equations and Optimization should lean heavily on applied problems. Computational methods should be stressed throughout. Further, as much interconnection as possible should be built into the whole program. It is planned that both the mathematics and the engineering courses should reinforce one another to an unprecedented degree.

A corollary of this requirement is that the courses should be taught by whoever is capable of doing the best job, regardless of the department he happens to be in. For the more standard and the more theoretical courses the mathematics department would be the natural place to look for teachers, but where applications are heavily stressed the best teacher may well be found in some other field.

Beyond a fairly standard 15-18 hour introduction to physics and chemistry the program calls for twelve more hours in the basic sciences. Six of these are accounted for by a mechanics course, intended to be a coordinated combination of physics, mathematics, and computing. In addition to the standard mechanics of particles and rigid bodies considerable time is spe[illegible] on variational methods and continuum mechanics.

The remaining six hours is divided between electromagnetics and thermodynamics (including statistical mechanics). Of the many possible continuations of the basic material it was felt that these two, because of their fundamental nature, their wide applicability, and their susceptibility to interesting mathematical analysis, were particularly appropriate to this program.

Modern computing facilities and the techniques for using them are still developing with bewildering rapidity, and no program fixed now will give adequate coverage for very long. We are painfully aware of these rapid developments and claim no special powers of prophecy. The proposal delineated here provides two realistic approaches to current problems of computing. A direct approach is the inclusion of two courses devoted to computation, the Numerical Analysis mentioned above and an Introduction to Computer Science of the type described in the CUPM Recommendations on the Undergraduate Mathematics Program for Work in Computing, the ACM Undergraduate Program (Communications of the Association for Computing

Machinery, 8 (1965) 543-552), or in the forthcoming revision of CUPM's Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists. These give the principles of modern computation, including the use of a programming language, and their basic applications to mathematical problems.

Less direct, but perhaps of more ultimate importance, is the inclusion of computational methods in connection with each appropriate topic in other courses. Such topics occur in almost all the core courses, but particularly in Differential Equations (both Ordinary and Partial), Mechanics, and Optimization. It is expected that significant problems to program and run on a computer will be part of the work in these courses. Only in this way can a real understanding of the power and (especially) of the limitations of modern computing techniques be communicated.

The overall structure of the core can be seen in the adjacent table. Here each entry represents a three-hour course.

THE CORE

Year	Core Courses		Electives*
I	Calculus	Physics Chemistry	6
	Calculus	Physics Chemistry	6
II	Linear Algebra	Physics Computer Science	6
	Calculus	Physics	9
III	1. Functions of Several Variables	3. Mechanics 4. Numerical Analysis 5. Probability	3
	2. Ordinary Differential Equations	3. Mechanics 5. Statistics	6
IV	6. Complex Variables	8. Electromagnetics	9
	7. Functional Analysis	9. Thermodynamics	9
V	10. Partial Differential Equations	11. Optimization	9
	10. Partial Differential Equations		12

*Semester hours

Of an assumed total of 150 semester hours for the five years, 45 hours are devoted to mathematics and computing, and 30 to basic sciences. This leaves the remaining 75 hours for humanities electives and for additional courses in engineering, mathematics, and science. With a roughly even split this should satisfy normal requirements. The most demanding of our sample options, Control Theory, specifies only 24 hours, leaving, say, 15 hours for basic engineering and technical electives, and 36 hours for the humanities.

The first two years of the program are fairly standard. For the mathematics portion we recommend the sequence of courses described in the CUPM report A General Curriculum in Mathematics for Colleges (1965). In addition to linear algebra and the usual elementary calculus of one or more variables this includes an introduction to differential equations.

Introductory physics and chemistry courses are at present under intensive review by professional groups; some radically new physics courses have recently appeared and other experimental programs are under way. We therefore refrain from specifying these courses in any detail but urge the reader to consult the publications of the Commission on College Physics and the Advisory Council on College Chemistry. Widespread adoption of new elementary courses could require great change in the content, or even in the selection, of the later science courses in the core.

The introductory computer science course should include a discussion of the nature of an automatic computer and the manner in which it solves problems, an introduction to a specific computer language and its role in this process, and some practice in the actual solution of various types of problems. Either by means of this course or from supplementary instruction, students should be able to program and run simple problems early in their sophomore year.

The third year has the heaviest concentration of core courses, foundation for the more advanced material in the core and for the technical applications in the options. These courses are of fairly standard type except for Mechanics, which has been described, and Intermediate Ordinary Differential Equations. The occurrence of considerable material on differential equations in the calculus course justifies the initial adjective in the latter title and permits the course to concentrate on linear equations with variable coefficients, boundary value problems, and special functions. There is also a brief introduction to nonlinear equations.

The report A General Curriculum in Mathematics for Colleges outlines two courses in Functions of Several Variables (there called Multivariable Calculus). One of these has a classical vector analysis approach while the other uses differential forms. We recommend the latter, partly because

the vector technique is covered in the physics courses but also because the more general approach is a valuable background for fourth year Functional Analysis.

With the exception of the Probability and Statistics all the third year work is closely interconnected, and considerable thought should be given to the sequence of topics so as to get the most coordination. In particular, linear algebra, numerical techniques, and the use of a computer to solve problems are ever-recurring themes in the year's work.

The fourth and fifth years of the core are fairly light, since here will come most of the work in the options. In the mathematics courses, in addition to the obvious requirements of Complex Variables and Partial Differential Equations—six hours of the latter is necessary for any sort of coverage—we have included courses in Functional Analysis and Optimization. The first of these provides an introduction to some hitherto abstract topics that are proving useful in a variety of applications, such as numerical analysis, communication theory, quantum physics, and many branches of mathematics. General metric and linear spaces, operators and functionals, with an introduction to measure theory, are the central topics.

Optimization is another introductory course, but one tied very closely to applications. Based on the notions of compactness, convexity, and Lagrange multipliers it treats briefly the various types of mathematical programming, some combinatorial problems, and the calculus of variations.

The two science courses in the fourth year, Electromagnetics and Thermodynamics, could vary considerably in content. In any case, however they should take advantage of the students' exceptional background in analysis, probability, mechanics, and computing to give a considerably more sophisticated treatment than could commonly be contemplated.

The selection and arrangement of the courses comprising the core represent the Panel's best judgment of the curriculum currently needed to develop the kind of highly trained but still flexible engineer described in the Introduction. Local conditions and opinions will undoubtedly suggest some changes, and future developments—for example, an upsurge of biological engineering—may call for a reappraisal of the whole program. But the basic framework of mathematics, science, and computing should still be appropriate for many years to come.

THE OPTIONS

The role of the option is to provide the student with a solid acquaintance with some branch of engineering, at the same time giving background and applications of many of the subjects treated in the core. In general, serious work in the option will begin in the fourth year, following the heavy load of third year core courses and some appropriate technical introduction. With this background the work in the option can begin, and proceed, at a higher level of sophistication than is usually possible.

As samples of what might be done we present three options, labeled, for want of better names, Operations Research in Systems Engineering, Orbit Mechanics, and Control Theory. There is nothing special about these; they simply happened to be topics of interest to some of the Panel members and consultants. Other topics of equal suitability might be, for example, Fluid Mechanics, Solid Mechanics (See Liebowitz and Allen, Editors, CURRICULA IN SOLID MECHANICS, Prentice-Hall, Inc., 1961), Electronics and Microwaves, Wave Propagation and Plasma Physics, Materials Engineering, and Nuclear Engineering.

For each of the sample options we give here a brief description of the program and an outline of its structure. Detailed syllabi of the courses are given in the Appendix.

OPERATIONS RESEARCH IN SYSTEMS ENGINEERING

Operations Research has been described as the application of mathematical methods to the solution of practical optimization problems in engineering, in business, and in government. The Operations Research Option builds onto the core those features of operations research that pertain especially to the design, development, and production of large-scale engineering systems. These require analysis of the complicated interrelationships among component and system performances, development and production costs, scheduling priorities, available manpower and facilities, and a host of other factors. Such considerations have made necessary the use of various optimization techniques, the application of probabilistic and statistical methods, the development of a highly mathematical reliability theory, Monte Carlo simulation methods, optimal control theory, linear, nonlinear and dynamic programming methods, and queueing theory. These mathematical topics provide the typical tools for operations research studies which find wide applicability in the evaluation (and comparison) of performance, programs, and policies in certain types of engineering and industrial situations.

The Operations Research Option, which has been fleshed out in some detail, represents an attempt to provide suitable training for engineers who have to cope with such problems. Building upon the six-hour course in Probability and Statistics of the core, it provides a six-hour course in mathematical methods of reliability engineering. The course includes both probabilistic models of reliability problems and statistical techniques of reliability estimation. The introductory optimization course of the core is supplemented by a further six-hour course in linear programming techniques, dynamic programming, inventory and scheduling problems, queueing theory, and related topics.

The third course recommended in this option is a three-hour course in System Simulation, which exploits the use of a computer in carrying out the analysis of such operations research activities.

Additional courses in economics, such as Economic Decision Theory, or in management science would constitute appropriate electives for certain students.

Note that the core course in Optimization has been moved into the fourth year to provide the necessary background for the fifth year course in Operations Research.

OPERATIONS RESEARCH IN SYSTEMS ENGINEERING

Year	Core Courses	Option Courses	Electives
IV	6. Complex Variables	OR1. Reliability	9
	7. Functional Analysis	OR1. Reliability	
	8. Electromagnetics		
	9. Thermodynamics and Statistical Mechanics		
	11. Optimization		
V.	10. Partial Differential Equations	OR2. Operations Research	15
	10. Partial Differential Equations	OR2. Operations Research	
		OR3. Systems Simulation	

ORBIT MECHANICS

As with each of the options, the aim is to build upon the foundation supplied by the core to provide greater specialization in an aspect of mathematics of central importance in modern engineering, here space science.

The design of space vehicles, prediction, correction and control of their space flight, transmission and evaluation of information collected in space—all such tasks place unusual new requirements on engineering skills and training. Additional problems arise from the necessity for real-time computations and corrections during space flight. Underlying all these difficulties is the problem of developing a correct physical intuition for the nature of space travel, vehicle control, and environmental conditions in space.

The Orbit Mechanics Option supplements the core courses in mechanics with substantial one-semester courses in celestial mechanics and in orbit theory. The addition of an advanced programming course and an introduction to control theory provide solid grounding for many problems of space vehicle engineering. The course in data smoothing and prediction provides training essential to the successful collection, retrieval, and interpretation of telemetered information.

Additional courses in astronomy or in space physics constitute natural electives for students in such a program.

Since some of Partial Differential Equations is needed for Advanced Numerical Analysis and Celestial Mechanics the core course 10 must be put in the fourth year. This gives a rather heavy concentration of mathematics in the fourth year, but this could be relieved, if desired, by a further shifting of some of the other courses.

ORBIT MECHANICS

Year	Core Courses	Option Courses	Electives
IV	6. Complex Variables 7. Functional Analysis 8. Electromagnetics 9. Thermodynamics and Statistical Mechanics 10. Partial Differential Equations 10. Partial Differential Equations	OM1. Advanced Numerical Analysis	9
V	11. Optimization	OM2. Advanced Programming OM3. Celestial Mechanics OM4. Orbit Theory CT2. Control CT5. Data Smoothing and Prediction	12

CONTROL THEORY

The advances in computers and in instrumentation have brought an enormous increase in the sophistication of control systems. The instruments allow us to measure rapidly and precisely many variables which were previously hard to measure and the computer allows us to make use of all the data while it is still current. The space program has given a great impetus to control theory by bringing up a number of new problems with very strict requirements. Another aspect of many control problems is that they involve control loops which extend over great distances, thereby creating an interface problem between the control and the communications specialist.

The Control Theory Option starts in the third year with a one semester course in circuit theory which exposes the student to the modeling problem, to some specific physical devices which he will encounter later and to basic system concepts in simple physical situations. In the fourth year the control course will furnish the student the basic facts about control systems and the linear systems course will provide the common base for further courses in control, communications, and circuits. The fifth year includes a course on the techniques of optimization, one on advanced control, one on advanced communications, and one on information theory.

CONTROL THEORY

Year	Core Courses	Option Courses	Electives
IV	6. Complex Variables	CT2. Control	6
	7. Functional Analysis	CT3. Laboratory	
	8. Electromagnetics	CT4. Linear Systems	
	9. Thermodynamics and Statistical Mechanics	CT5. Data Smoothing and Prediction	
V	10. Partial Differential Equations	CT6. Advanced Control	12
	10. Partial Differential Equations	CT7. Information Theory	
	11. Optimization	CT8. Advanced Communications	

APPENDIX

Sample Outlines of the Courses

The following course outlines are intended in part as extended expositions of the ideas that we have in mind, in part as feasibility studies, and in part as proposals for the design of courses and textbooks. They have a wide variety of origins. Some are standard courses now given in universities and some are experiments that have never yet been tried. Most of them, however, are modifications or combinations, more or less radical, of familiar material. They have been prepared by many different persons, with a broad spectrum of interests in mathematics and related fields, and representing industrial as well as academic interests. However, all outlines were carefully scrutinized by the whole Panel, and were not accepted until their value to the whole program was clear. For better or worse, this is a committee product.

As with the outlines, the references vary considerably in style. In most cases they are not intended as texts but as indications of possible sources of material to be moulded into the course.

It will be observed that there is considerable overlapping in some of the content of the courses, for example in Intermediate Ordinary Differential Equations and in Numerical Analysis. This is inevitable in the courses in any modern university, where most courses are taken by a variety of students in different programs and with different backgrounds. If a neater dovetailing of these courses is possible in particular cases, the contents should of course be modified accordingly.

CONTENTS

THE CORE

I, II	Physics	(12)	Calculus and Linear Algebra	(12-15)
	Chemistry	(6)	Computer Science	(3)

III	1. Functions of Several Variables	(3)
	2. Intermediate Ordinary Differential Equations	(3)
	3. Mechanics	(6)
	4. Numerical Analysis	(3)
	5. Probability and Statistics	(6)
IV	6. Complex Variables	(3)
	7. Functional Analysis	(3)
	8. Electromagnetics	(3)
	9. Thermodynamics and Statistical Mechanics	(3)
V	10. Partial Differential Equations	(6)
	11. Optimization	(3)

1. Functions of Several Variables (3 semester hours).

A study of the properties of continuous mappings from E_n to E_m, making use of linear algebra, and an introduction to differential forms and vector calculus, based upon line integrals, surface integrals, and the general Stokes' theorem. Application should be made to field theory, or to elementary hydrodynamics, or other similar topics, so that some intuitive understanding can be gained.

a. Transformations (15 lessons). Functions (mappings) from E_n to E_m, for n, m = 1, 2, 3, 4. Continuity and implications of continuity; differentiation, and the differential of a mapping as a matrix valued function. The role of the Jacobian as the determinant of the differential; local and global inverses of mappings, and the implicit function theorem. Review of the chain rule for differentiation, and reduction to matrix multiplication. Application to change of variable in multiple integrals, and to the area of surfaces.

b. Differential forms (6 lessons). Integrals along curves. Introduction of differential forms; algebraic operations; differentiation rules. Application to change of variable in multiple integrals. Surface integrals, and the meaning of a general k-form.

c. Vector analysis (4 lessons). Reinterpretation of differential forms in terms of vectors; vector function as mapping into E_3; vector field as mapping from E_3 into E_3. Formulation of line and surface integrals (1-forms and 2-forms) in terms of vectors. The operations Div, Grad, Curl, and their corresponding translations into differential forms.

d. Vector Calculus (8 lessons). The theorems of Gauss, Green, Stokes, stated for differential forms, and translated into vector equivalents. Invariant definitions of Div and Curl. Exact differential forms, and independence of path for line integrals. Application to a topic in hydrodynamics, or to Maxwell's equations, or to the derivation of Green's identities and their specializations for harmonic functions.

e. Fourier methods (6 lessons). The continuous functions as a vector (linear) space; inner products and orthogonality; geometric concepts and analogy with E_n. Best L^2 approximation; orthogonal bases, completeness, Schwarz and Bessel inequalities. General Fourier series with respect to an orthonormal basis. Treatment of the case $\{e^{inx}\}$ and the standard trigonometric case. Application to the solution of one standard boundary value problem.

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2. Intermediate Ordinary Differential Equations (3 semester hours).

The presentation of the course material should include: (1) an account of the manner in which ordinary differential equations and their boundary value problems, both linear and nonlinear, arise; (2) a carefully reasoned discussion of the qualitative behavior of the solution of such problems, sometimes on a predictive basis and at other times in an *a posteriori* manner; (3) a clearly described awareness of the role of numerical processes in the treatment of these problems, including the disadvantages as well as advantages—in particular, there should be a firm emphasis on the fact that numerical integration is not a substitute for thought; (4) an admission that we devote most of our lecture time to linear problems because (with isolated exceptions) we don't know much about any nonlinear ones except those that (precisely or approximately) can be attacked through our understanding of the linear ones. Thus a thorough treatment of the linear problems should precede a sophisticated attack on the nonlinear ones.

The distribution of time between items d and e cannot be prescribed easily or with universal acceptability. Only a superficial account of these topics can be given in the available time, but each should be introduced.

a. Systems of linear ordinary differential equations with constant coefficients (6 lessons). Review of homogeneous and nonhomogeneous problems; superposition and its dependence on linearity; transients in mechanical and electrical systems. The Laplace transform as a carefully developed operational technique without inversion integrals.

b. Linear ordinary differential equations with variable coefficients (10 lessons). Singular points, series solutions about regular points and about singular points. Bessel's equation and Bessel functions, Legendre's equation and Legendre polynomials, confluent hypergeometric functions. Wronskians, linear independence, number of linearly independent solutions of an ordinary differential equation. Sturm-Liouville theory and eigenfunction expansions.

c. Solution of boundary value problems involving nonhomogeneous linear ordinary differential equations (7 lessons). Methods using Wronskiar Green's functions (introduce δ function), and eigenfunction expansions. Numerical methods. Rudimentary existence and uniqueness questions.

d. Asymptotic expansion and asymptotic behavior of solutions of ordinary differential equations (3 lessons). Essentially the material on pp. 468-470 and pp. 488-495 of Jeffreys and Jeffreys [8].

e. Introduction to nonlinear ordinary differential equations (6 lessons) Special nonlinear equations which are reducible to linear ones or to quadratures, elliptic functions (pendulum oscillations), introductory phase plane analysis (Poincaré).

f. Numerical methods (7 lessons). Step-by-step solution of initial value problems for single equations and for systems. Error analysis, roundoff, stability. Improper boundary conditions, discontinuities, and other pitfalls.

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2. Brenner, J. L. PROBLEMS IN DIFFERENTIAL EQUATIONS. San Francisco: W. H. Freeman, 1963.

3. Brouwer, D.,and Clemence, G. M. METHODS OF CELESTIAL MECHANICS. New York: Academic Press, 1961.

4. Coddington, E. AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS. Englewood Cliffs, New Jersey: Prentice-Hall, 1961.

5. Crandall, S. H. ENGINEERING ANALYSIS. New York: McGraw-Hill Book Co., 1956.

6. Erdélyi, A. ASYMPTOTIC EXPANSIONS. New York: Dover Publications, 1956.

7. Ince, E. L. ORDINARY DIFFERENTIAL EQUATIONS. New York: Dover Publications, 1944.

8. Jeffreys, H., and Jeffreys, B. S. METHODS OF MATHEMATICAL PHYSICS. New York: Cambridge University Press, 1956, third edition.

9. Kaplan, W. ORDINARY DIFFERENTIAL EQUATIONS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

10. Pontryagin, L. S. ORDINARY DIFFERENTIAL EQUATIONS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

11. Tricomi, F. G. DIFFERENTIAL EQUATIONS. New York: Hafner Publishing Co., 1961.

3. Mechanics (6 semester hours).

The course outlined below differs from that given in such a book as H. Goldstein's CLASSICAL MECHANICS in that the discussion of mechanics is interrupted at various stages in order to deal with topics in numerical analysis; e.g., after the equations of motion are formulated various methods for numerically integrating initial value problems are discussed and analyzed. It is assumed that the student has had the linear algebra course as well as the computer science course. Homework assignments that involve use of a computer should be made.

a. Kinematics (8 lessons). Cartesian coordinates in Euclidean three space, cartesian tensors, the numerical tensors δ_{ij}, e_{ijk}. Parametric equations of curves. Velocity and acceleration in cartesian coordinates, in general coordinates. Moving general coordinates and the velocity and acceleration in such coordinates. Equations of straight lines in moving general coordinates. Characterization of inertial coordinate frames.

b. Particle mechanics (10 lessons). Equations of motion. Initial value problems for a system of ordinary differential equations, existence, uniqueness, continuous dependence on parameters and initial values. Numerical methods for integrating initial value problems, their stability.

c. Perturbation theory (8 lessons). Physical stability. Numerical stability. Linearization of nonlinear problems.

d. Central forces (10 lessons). Planetary orbits. Energy integrals,

angular momentum integrals. Constants of motion and symmetry properties.

e. Variational principles and rigid body motion (13 lessons). Hamilton's principle, generalized coordinates of Lagrange, canonical equations, contact transformations, partial differential equations of Hamilton and Jacobi. Rigid body motion.

f. Multidimensional variational principles (8 lessons). Variation of multiple integrals and applications to problems in statics and dynamics of deformable bodies. Vibrating strings and membranes. Rayleigh-Ritz method. Use of polynomials to derive difference equation approximation to the boundary value differential equations that are the Euler equations of a variational principle. Numerical integration of boundary value problems on the line and in the plane.

g. Continuum mechanics (21 lessons). Stress and strain tensors. Conservation of mass, momentum and energy. Partial differential equations describing the motion of a perfect fluid. One dimensional isentropic motions (simple and compound waves). Numerical integration of one dimensional motions. Existence of shocks. Numerical integration in the presence of shocks.

REFERENCES

1. Courant, R., and Friedrichs, K. O. SUPERSONIC FLOW AND SHOCK WAVES. New York: John Wiley & Sons, 1948.

2. Fox, L. NUMERICAL SOLUTION OF TWO-POINT BOUNDARY PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS. New York: Oxford University Press, 1957.

3. Goldstein, H. CLASSICAL MECHANICS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1950.

4. Henrici, P. DISCRETE VARIABLE METHODS IN ORDINARY DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1962.

5. Landau, L. D., and Lifshitz, E. MECHANICS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1960.

6. Synge, J. L., and Griffith, B. PRINCIPLES OF MECHANICS. New York: McGraw-Hill Book Co., 1959, third edition.

4. Numerical Analysis (3 semester hours).

An introduction to the solution of mathematical problems by computation, including both finite and iterative methods and some error analysis. The algorithmic approach should be emphasized and problems should be programmed and run on a computer. Courses in computer science and linear algebra are prerequisites.

a. Solution of equations (9 lessons). Functional iteration of (non-linear) equations, including convergence theorems, error effects, Aitken's δ^2 acceleration; analysis of special methods such as the methods of false position and of Newton; iteration for systems of equations, methods of Bernoulli, Sturm, Bairstow, Muller, etc. for finding roots of polynomial equations.

b. Polynomial approximations; interpolation and quadrature (12 lessons). Weierstrass theorem, Bernstein polynomials, Lagrange interpolation with error formulas. Least squares, orthonormal systems relative to given weight functions; concept and analysis of best approximation relative to given criteria—Chebyshev polynomials, trigonometric approximations. Differencing, interpolation schemes, and formal difference calculus; quadrature formulas of the interpolation and Gaussian types with an analysis of error; Richardson acceleration, Romberg integration; numerical quadrature for improper integrals.

c. Initial value problems for ordinary differential equations (9 lessons). Reduction to first order systems, Runge-Kutta, Adams, predictor-

corrector methods, elementary considerations of stability and round-off.

d. Matrix inversion and matrix eigenvalues (9 lessons). Gaussian elimination and some iterative methods for inversion; application to elliptic differential equations; Rayleigh quotients and power methods for obtaining the eigenvalues of symmetric matrices; analysis of convergence.

REFERENCES

1. Faddeeva, V. N. COMPUTATIONAL METHODS OF LINEAR ALGEBRA. New York: Dover Publications, 1959.

2. Fox, L. INTRODUCTION TO NUMERICAL LINEAR ALGEBRA, WITH EXERCISES. New York: Oxford University Press, 1965.

3. Hamming, R. W. NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS. New York: McGraw-Hill Book Co., 1962.

4. Henrici, P. DISCRETE VARIABLE METHODS IN ORDINARY DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1962.

5. Henrici, P. ELEMENTS OF NUMERICAL ANALYSIS. New York: John Wiley & Sons, 1964.

6. Householder, A. S. PRINCIPLES OF NUMERICAL ANALYSIS. New York: McGraw-Hill Book Co., 1953.

7. Teddington, England. National Physical Laboratory. MODERN COMPUTING METHODS. London: H. M. Stationery Office, 1962, second edition.

8. Noble, B. NUMERICAL METHODS. New York: John Wiley & Sons, 1965, two volumes.

9. Ralston, A. FIRST COURSE IN NUMERICAL ANALYSIS. New York: McGraw-Hill Book Co., 1965.

10. Stiefel, E. L. AN INTRODUCTION TO NUMERICAL MATHEMATICS. New York: Academic Press, 1963.

11. Todd, J. SURVEY OF NUMERICAL MATHEMATICS. New York: McGraw-Hill Book Co., 1962.

12. Wendroff, B. THEORETICAL NUMERICAL ANALYSIS. New York: Academic Press, 1966.

5. Probability and Statistics (6 semester hours).

This is a one year course presenting the basic theory of probability and statistics. Although the development of the ideas and results is mathematically precise, the aim is to prepare students to formulate realistic models and to apply appropriate statistical techniques in problems likely to arise in engineering. Therefore new ideas will be motivated and applications of results will be given wherever possible.

First Semester: Probability.

a. Basic probability theory (4 lessons). Different theories of probability (classical, frequency, axiomatic). Combinatorial methods for computing probability. Conditional probability, independence. Bayes' theorem. Geometrical probability.

b. Random variables (5 lessons). Concept of random variable and of distribution function. Discrete and continuous types. Multidimensional random variables. Marginal and conditional distributions.

c. Parameters of a distribution (4 lessons). Expected values. Moments. Moment generating functions. Moment inequalities.

d. Characteristic functions (4 lessons). Definition, properties. Characteristic function and moments. Determination of distribution function from characteristic function.

e. Various probability distributions (6 lessons). Binomial, Poisson, multinomial. Uniform, normal, gamma, Weibull, multivariate normal. Importance of normal distribution. Applications of normal distribution to

error analysis.

f. Limit theorems (6 lessons). Various kinds of convergence. Law of large numbers. Central limit theorem.

g. Markov chains (4 lessons). Transition matrix. Ergodic theorem.

h. Stochastic processes (6 lessons). Markov processes. Processes with independent increments. Poisson process. Wiener process. Stationary processes.

Second Semester: Statistics.

a. Sample moments and their distributions (5 lessons). Sample, statistic. Distribution of sample mean. Student's distribution. Fisher's Z distribution.

b. Order statistics (4 lessons). Empirical distribution function. Tolerance limits. Kolmogorov-Smirnov statistic.

c. Tests of hypotheses (5 lessons). Simple hypothesis against simple alternative. Composite hypotheses. Likelihood ratio test. Applications.

d. Point estimation (5 lessons). Consistent estimates. Unbiased estimates. Sufficient estimates. Efficiency of estimate. Methods of finding estimates.

e. Interval estimation (6 lessons). Confidence and tolerance intervals. Confidence intervals for large samples.

f. Regression and linear hypotheses (4 lessons). Elementary linear models. The general linear hypothesis.

g. Nonparametric methods (5 lessons). Tolerance limits. Comparison of two populations. Sign test. Mann-Whitney test.

h. Sequential methods (5 lessons). The probability ratio sequential test. Sequential estimation.

PRINCIPAL REFERENCES

1. Birnbaum, Z. W. INTRODUCTION TO PROBABILITY AND MATHEMATICAL STATISTICS. New York: Harper & Row, 1962.

2. Feller, W. PROBABILITY THEORY AND ITS APPLICATIONS. New York: John Wiley & Sons, Volume I, 1957; Volume II, 1966.

3. Fisz, M. PROBABILITY THEORY AND MATHEMATICAL STATISTICS. New York: John Wiley & Sons, 1963, third edition.

4. Mood, A. M., and Graybill, F. A. INTRODUCTION TO THE THEORY OF STATISTICS. New York: McGraw-Hill Book Co., 1963, second edition.

ADDITIONAL SPECIALIZED REFERENCES

1. Kempthorne, O. THE DESIGN AND ANALYSIS OF EXPERIMENTS. New York: John Wiley & Sons, 1952.

2. Kendall, M. G., and Moran, P. A. P. GEOMETRICAL PROBABILITY. New York: Hafner Publishing Co., 1963.

3. Owen, D. B. HANDBOOK OF STATISTICAL TABLES. Reading, Massachusetts, Addison-Wesley Publishing Co., 1962.

4. Papoulis, A. PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES. New York: McGraw-Hill Book Co., 1965.

5. Parzen, E. MODERN PROBABILITY THEORY AND ITS APPLICATIONS. New York: John Wiley & Sons, 1960.

6. Pfeiffer, P. CONCEPTS OF PROBABILITY THEORY. New York: McGraw-Hill Book Co., 1965.

6. Functions of a Complex Variable (3 semester hours).

The development of skills in this area is very important in the sciences, and the course must exhibit many examples which illustrate the influence of singularities and which require varieties of techniques for finding conformal maps, for evaluating contour integrals (especially those with multivalued integrands), and for using integral transforms.

a. Introduction (4 lessons). The algebra and geometry of complex numbers. Definitions and properties of elementary functions (e.g., e^z, $\sin z$, $\log z$, z^ν).

b. Analytic functions (2 lessons). Limits, derivative, Cauchy-Riemann equations.

c. Integration (6 lessons). Integrals, functions defined by integrals. Cauchy's theorem and formula, integral representation of derivatives of all orders. Maximum modulus, Liouville's theorem, fundamental theorem of algebra.

d. Series (5 lessons). Taylor and Laurent series. Uniform convergence, term-by-term differentiation, uniform convergence in general. Domain of convergence and classification of singularities.

e. Contour integration (3 lessons). The residue theorem. Evaluation of integrals involving single valued functions.

f. Analytic continuation and multivalued functions (4 lessons). Analytic continuation, multivalued functions and branch points. Technique for contour integrals involving multivalued functions.

g. Conformal mapping (5 lessons). Conformal mapping. Bilinear and Schwarz-Christoffel transformations, use of mapping in contour integral evaluation. Some mention should be made of the general Riemann mapping theorem.

h. Asymptotic evaluation of integrals (3 lessons). The methods of steepest descent and stationary phase. This is a good place to develop a detailed picture of the properties of some of the special functions (e.g., Bessel and gamma functions).

i. Boundary value problems (3 lessons). Laplace's equation in two dimensions and the solution of some of its boundary value problems, using conformal mapping.

j. Integral transforms (4 lessons). The Fourier and Laplace transforms, their inversion identities, and their use in boundary value problems.

REFERENCES

1. Ahlfors, L. V. COMPLEX ANALYSIS. New York: McGraw-Hill Book Co., 1966, second edition.

2. Carrier, G. F., Krook, F., and Pearson, C. E. FUNCTIONS OF A COMPLEX VARIABLE, THEORY AND APPLICATIONS. New York: McGraw-Hill Book Co., 1966.

3. Churchill, R. V. COMPLEX VARIABLES AND APPLICATIONS. New York: McGraw-Hill Book Co., 1960, second edition.

4. Hille, E. ANALYTIC FUNCTION THEORY. Boston: Ginn & Co., Volume I, 1959; Volume II, 1962.

5. Knopp, K. THEORY OF FUNCTIONS. New York: Dover Publications, Volume I, 1945; Volume II, 1947.

6. Nehari, Z. CONFORMAL MAPPING. New York: McGraw-Hill Book Co., 1952.

7. Nehari, Z. INTRODUCTION TO COMPLEX ANALYSIS. Boston: Allyn and Bacon, 1961.

ADDITIONAL REFERENCES FOR SPECIAL TOPICS

1. DeBruijn, N. G. ASYMPTOTIC METHODS IN ANALYSIS. New York: John Wiley & Sons (Interscience), 1958.

2. Jeffreys, H., and Jeffreys, B. S. METHODS OF MATHEMATICAL PHYSICS. New York: Cambridge University Press, 1956, third edition.

3. Morse, P. M., and Feshbach, H. METHODS OF THEORETICAL PHYSICS. New York: McGraw-Hill Book Co., 1953.

4. Papoulis, A. THE FOURIER INTEGRAL AND ITS APPLICATIONS. New York: McGraw-Hill Book Co., 1962.

5. Sneddon, I. N. FOURIER TRANSFORMS. New York: McGraw-Hill Book Co., 1951.

6. Whittaker, E. T., and Watson, G. N. MODERN ANALYSIS. New York: Cambridge University Press, 1927, fourth edition.

7. Introduction to Functional Analysis (3 semester hours).

The purpose of this course is to present some of the basic ideas of elementary functional analysis in a form which permits their use in other courses in mathematics and its applications. It should also enable a student to gain insight into the ways of thought of a practicing mathematician and should open up much of the modern technical literature dealing with operator theory.

Prerequisite to this course is a good foundation in linear algebra, and in the concepts and techniques of the calculus of several variables. The material of this course should be presented with a strong geometrical flavor; undue time should not be spent on the more remote and theoretical aspects of functional analysis. Topics should be developed and first employed in mathematical surroundings familiar to the student. It would be very much in keeping with the intention of the course to emphasize the relationship between functional analysis and approximation theory, discussing (for example) some aspects of best uniform or best L^2 approximation to functions, and some error estimates in integration or interpolation formulas.

While some knowledge of measure theory and Lebesgue integration is needed for an adequate presentation of this material, it is not intended that the treatment be as complete as that in a standard real analysis course. The intended level is that to be found in the treatment by Kolmogorov and Fomin [5] or perhaps that in Chapters III and IV of [9]. If there is additional time, students might be introduced to some of the elementary

theory of integral equations, or to applications in probability theory, or to the study of a specific compact operator, or to distributions ([4], [7], [8]).

a. Metric spaces (9 lessons). Basic topological notions, mappings and continuity, complete metric spaces; examples. The fixed point theorem for contraction mappings, and applications (e.g., the initial value problem for ordinary differential equations, nonlinear integral equations, the implicit function theorem). Study of the space of complex valued functions on a compact metric space. Equicontinuity, and Arzela's theorem, with applications. The Stone-Weierstrass theorem.

b. Normed linear spaces (9 lessons). Examples, including sequence spaces. Hölder and Minkowski inequalities. Linear functionals. The Hahn Banach theorem, and the principle of uniform boundedness. The dual space of a normed linear space, and representations of continuous functions on various sequence spaces. Weak convergence. Linear operators on normed linear spaces, boundedness and continuity; examples. Iteration methods for solving linear systems; inversion of linear operators which are close to the identity; examples. The spectrum and resolvent of an operator.

c. Hilbert space (7 lessons). Inner product spaces, orthogonality; projections and subspaces (should be used as a concrete example). Representation of continuous functionals. Orthonormal systems, and completeness (closure). Fourier expansion, Bessel's inequality. Examples of continuous linear operators; the adjoint of a linear operator.

d. Measure theory (14 lessons). Lebesgue measure on the line. Measurable functions; convergence a.e. The Lebesgue integral; the monotone, bounded and dominated convergence theorems. L^2 as a Hilbert space. Convergence (and summability) of Fourier series. Point measures; general measures. The Riesz representation theorem. Compact operators and the Fredholm alternative. Spectral decomposition for a self adjoint compact operator.

REFERENCES

1. Buck, R. C. (Editor). STUDIES IN MODERN ANALYSIS. Buffalo: Mathematical Association of America, 1962.

2. Davis, P. J. INTERPOLATION AND APPROXIMATION. New York: Blaisdell Publishing Co., 1963.

3. Goffman, C., and Pedrick, G. FIRST COURSE IN FUNCTIONAL ANALYSIS. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

4. Halperin, I., and Schwartz, L. INTRODUCTION TO THE THEORY OF DISTRIBUTIONS. Toronto, Canada: University of Toronto Press, 1952.

5. Kolmogorov, A. N., and Fomin, S. V. MEASURE, LEBESGUE INTEGRALS AND HILBERT SPACE. New York: Academic Press, 1961.

6. Lyusternik, L., and Sobolev, V. ELEMENTS OF FUNCTIONAL ANALYSIS. New York: Frederick Ungar Publishing Co., 1961.

7. Lorch, E. R. SPECTRAL THEORY. New York: Oxford University Press, 1962.

8. Riesz, F., and Sz-Nagy, B. FUNCTIONAL ANALYSIS. New York: Frederick Ungar Publishing Co., 1955.

9. Royden, H. L. REAL ANALYSIS. New York: Macmillan Co., 1963.

10. Rudin, W. PRINCIPLES OF MATHEMATICAL ANALYSIS. New York: McGraw-Hill Book Co., 1964, second edition.

11. Vulikh, B. Z. INTRODUCTION TO FUNCTIONAL ANALYSIS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1963.

12. Williamson, J. H. LEBESGUE INTEGRATION. New York: Holt, Rinehart and Winston, 1962.

8. Electromagnetics (3 semester hours).

This course combines the essentials of classical electromagnetic theory with the foundations of applications to plasma media. It should take advantage of the preparation in mechanics, especially continuum mechanics, as well as fundamentals of electricity and magnetism in the introductory physics course.

a. Electrostatics (4 lessons). Vacuum field and potential theorems. Dielectrics. Boundary conditions. Energy relations and forces.

b. Magnetic fields (5 lessons). Currents and moving charges. The vector potential. Magnetic media. Energy relations and forces.

c. Maxwell's equations (6 lessons). The law of induction. Maxwell's equations, extended to moving media. Energy, force and momentum relations in the electromagnetic field.

d. Wave propagation (8 lessons). The scalar wave equation. Plane, cylindrical, spherical waves. Homogeneous isotropic media. Dispersion. Nonhomogeneous isotropic media. Rays; the geometrical optics approximation. Wave packets, including nonhomogeneous media and absorption.

e. Electromagnetic waves (7 lessons). Free space and homogeneous isotropic media. Homogeneous plasmas. Inhomogeneous media. Anisotropic media, including plasma with magnetic field.

f. Radiation (9 lessons). Simple radiating systems. Radiation by moving charges. Radiation in ionized gases. Synchrotron radiation.

REFERENCES

1. Brandstatter, J. J. INTRODUCTION TO WAVES, RAYS, AND RADIATION IN PLASMA MEDIA. New York: McGraw-Hill Book Co., 1963.

2. Corson, D. R., and Lorrain, P. INTRODUCTION TO ELECTROMAGNETIC FIELDS AND WAVES. San Francisco: W. H. Freeman and Co., 1962.

3. Gartenhaus, S. ELEMENTS OF PLASMA PHYSICS. New York: Holt, Rinehart and Winston, 1964.

4. Ginsburg, V. L. PROPAGATION OF ELECTROMAGNETIC WAVES IN PLASMA. New York: Gordon and Breach, 1961.

5. Jackson, J. D. CLASSICAL ELECTRODYNAMICS. New York: John Wiley & Sons, 1962.

6. Kip, A. F. FUNDAMENTALS OF ELECTRICITY AND MAGNETISM. New York: McGraw-Hill Book Co., 1962.

7. Landau, L. D., and Lifshitz, E. ELECTRODYNAMICS OF CONTINUOUS MEDIA. Reading, Massachusetts: Addison-Wesley Publishing Co., 1960.

8. Lehnert, B. P. DYNAMICS OF CHARGED PARTICLES. New York: John Wiley & Sons, 1964.

9. Commission on College Physics, Plasma Physics Study Group (S. C. Brown, Chairman). "Outline of a course in Plasma Physics." AMERICAN JOURNAL OF PHYSICS 31(1963), pp. 637-691.

10. Panofsky, W. K. H., and Phillips, M. CLASSICAL ELECTRICITY AND MAGNETISM. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962, second edition.

11. Reitz, J. R., and Milford, F. J. FOUNDATIONS OF ELECTROMAGNETIC THEORY. Reading, Massachusetts: Addison-Wesley Publishing Co., 1960.

12. Stix, T. THEORY OF PLASMA WAVES. New York: McGraw-Hill Book Co., 1962.

9. Thermodynamics and Statistical Mechanics (3 semester hours).

There is a current trend to combine the macroscopic and the microscopic aspects of thermal physics from the beginning, instead of giving a careful treatment of classical thermodynamics, with applications, as in THERMAL PHYSICS by Morse. The suggested outline follows the latter plan, as probably more appropriate as a background for varied applications.

Thermodynamics.

a. State variables and equations of state (3 lessons). Temperature, pressure, heat and energy. Extensive and intensive variables. Pairs of mechanical variables. The perfect gas and other equations of state.

b. The first law of thermodynamics (4 lessons). Work, internal energy, heat. Heat capacities. Isothermal and adiabatic processes.

c. The second law of thermodynamics (6 lessons). Heat cycles. Reversible and irreversible processes. Entropy. Applications to simple thermodynamic systems.

d. The thermodynamic potentials (3 lessons). Internal energy, enthalpy, Gibbs and Helmholtz potentials. Examples and procedures for calculation.

e. Phase equilibria (3 lessons). Melting, evaporation, triple point and critical point.

f. Chemical applications (2 lessons). Reaction heats, electrochemical processes.

Statistical Mechanics (Equilibrium).

a. Statistical methods (3 lessons). Random walk; probability distributions; mean values; binomial, Poisson, and Gaussian distributions.

b. Statistical description of systems of particles (3 lessons). Ensembles, ergodic hypothesis, postulates, limiting behavior for large N , fluctuations.

c. Quantum statistics (5 lessons). Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distributions with applications (solids, gases, electron gas, blackbody radiation, etc.).

Microscopic Description of Non-Equilibrium.

a. Elementary kinetic theory (2 lessons).

b. Transport theory (2 lessons). Based on Boltzmann's equation, in simplified form.

c. Brownian motion (3 lessons). Possibly including the Fokker-Planck equation. More on random variables. Markov processes, fluctuations, irreversible processes.

REFERENCES

1. Crawford, F. H. HEAT, THERMODYNAMICS AND STATISTICAL PHYSICS. New York: Harcourt, Brace and World, 1963. The exposition of classical thermodynamics is similar to Zemansky's but, in addition, there is quite a good introductory coverage of statistical theory.

2. Lewis, G. N., and Randall, M. THERMODYNAMICS. New York: McGraw-Hill Book Co., 1961, second edition by Pitzer, K. S., and Brewer, L. A valuable reference for acquaintance with the reality of measuring thermal quantities.

3. Sears, F. W. THERMODYNAMICS, THE KINETIC THEORY OF GASES AND STATISTICAL MECHANICS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1953, second edition. Parts of this text are excellent; in general, the discussions are more elementary than those in Crawford's book.

4. Zemansky, M. W. HEAT AND THERMODYNAMICS. New York: McGraw-Hill Book Co., 1957, fourth edition. A standard and very good introduction to classical thermodynamics. Virtually no statistics.

More Advanced Treatments of Macroscopic Thermodynamics.

1. Callen, H. B. THERMODYNAMICS. New York: John Wiley & Sons, 1960. Theoretical macroscopic treatment, excellent but too formal to serve as course text at this level.

2. Chandrasekhar, S. INTRODUCTION TO THE STUDY OF STELLAR STRUCTURE. (Chapters 1 and 2). New York: Dover Publications, 1939. Brief and formal treatment following Caratheodory's axiomatic point of view.

3. Denbigh, K. G. THERMODYNAMICS OF THE STEADY STATE. New York: John Wiley & Sons, 1951. By far the best introduction to non-equilibrium thermodynamics including carefully chosen examples.

4. Epstein, P. S. TEXTBOOK OF THERMODYNAMICS. New York: John Wiley & Sons, 1937. A well-written older book, but the notation is often difficult to follow, so it is unsuitable as a primary textbook. Many of the applications especially are useful. A concise and interesting history of the First Law is included.

5. Pippard, A. B. THE ELEMENTS OF CLASSICAL THERMODYNAMICS. New York: Cambridge University Press, 1957. Not a textbook, but a marvelous short book emphasizing difficult points often obscured in standard works.

Treatments of Both Macroscopic and Microscopic Thermodynamics.

1. King, A. L. THERMOPHYSICS. San Francisco: W. H. Freeman and Co., 1962. A less ambitious work than the following textbooks.

2. Morse, P. M. THERMAL PHYSICS. New York: W. A. Benjamin, 1964, revised edition. This is based on a course similar to that outlined here. Parts are imaginatively done and a number of excellent problems are included. It is perhaps the best textbook presently available for at least the first part of a course such as is suggested here, except that the treatment is very condensed.

3. Reif, F. FUNDAMENTALS OF STATISTICAL AND THERMAL PHYSICS. New York: McGraw-Hill Book Co., 1965. Like Morse's book, this text is based on a course, but also contains much additional material. The macroscopic and microscopic aspects are more intimately combined throughout. An outstanding feature is the most ambitious treatment of non-equilibrium statistical mechanics appearing in a textbook intended for use at this level.

10. Partial Differential Equations (6 semester hours).

This course ordinarily occurs in the fifth year of the sequence, although with certain options (Orbit Mechanics,...) it should be taken in the fourth year. The material should strike a reasonable balance between the classical analytical theory of partial differential equations and modern computational aspects of the subject. For that reason existence theorems and the like should be of the constructive type whenever possible. Further, application to problems in classical and modern physics should constantly be borne in mind. Physical models should be used both to predict results concerning the behavior of solutions to partial differential equations and to interpret phenomena revealed analytically or computationally.

a. Introduction (6 lessons). Derivation of some equations, discussion of mathematical models, continuous dependence theorems, and relation to physics.

b. Classification and characteristics (9 lessons). Cauchy problem for first order equations, formulation and statement of Cauchy-Kowalewski theorem.

c. Hyperbolic equations (12 lessons). Existence and continuous dependence for second order equations. Riemann method. Three-dimensional wave equation. Retarded potentials. Numerical methods—finite difference schemes and stability.

d. Elliptic equations (21 lessons). Potential theory in three dimensions with smooth boundaries. Eigenvalue problems—estimates. Numerical methods.

e. Parabolic equations (12 lessons). Thermal potential theory. Convergence to steady state and relation to potential problems. Numerical methods, and connection, in steady state, to numerical methods for elliptic problems.

f. Integral representation of solutions (12 lessons). Green's functions. Integral equations.

g. Equations of hydrodynamics (6 lessons). Shock phenomena, weak solutions. Numerical methods.

REFERENCES

1. Courant, R., and Friedrichs, K. O. SUPERSONIC FLOW AND SHOCK WAVES. New York: John Wiley & Sons, 1948.

2. Courant, R., and Hilbert, D. METHODS OF MATHEMATICAL PHYSICS. New York: John Wiley & Sons, Volume I, 1953; Volume II, 1962.

3. Forsythe, G. E., and Wasow, W. R. FINITE DIFFERENCE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1960.

4. Garabedian, P. R. PARTIAL DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1964.

5. Richtmyer, R. D. DIFFERENCE METHODS FOR INITIAL-VALUE PROBLEMS. New York: John Wiley & Sons, 1958.

6. Sommerfeld, A. PARTIAL DIFFERENTIAL EQUATIONS IN PHYSICS. New York: Academic Press, 1949.

7. Tikhonov, A. N., and Samarski, A. A. EQUATIONS OF MATHEMATICAL PHYSICS. New York: Pergamon Press, 1964.

11. Optimization (3 semester hours).

Attempts to determine the "best" or "most desirable" solution to large-scale engineering problems inevitably lead to optimization studies. Generally, the appropriate methods are highly mathematical and include such relatively new techniques as mathematical programming, optimal control theory, and certain combinatorial methods, in addition to more classical techniques of the calculus of variations and standard maxima-minima considerations of the calculus.

This three semester hour course in optimization is planned to provide a basic mathematical background for such optimization studies. It will, in addition, acquaint the student with references and illuminate directions for further study. A natural successor to this course is the six semester hour course, Operations Research, in the Operations Research Option, where linear programming techniques are developed in depth and additional topics in dynamic programming, inventory and scheduling problems, Monte Carlo simulation techniques, and queueing theory are introduced.

a. Simple, specific examples of typical optimization problems (3 lessons). Minimization with side conditions (Lagrange multipliers, simple geometrical example). Linear program (diet problem). Nonlinear program (least squares under inequality constraints, delay line problem). Combinatorial problem (marriage or network). Variational problem (brachistochrone). Control problem (missile). Dynamic program (replacement schedule).

b. Convexity and n-space geometry (6 lessons). Convex regions, functions, general definition (homework: use definition to show convexity (or nonconvexity) in nonobvious cases, as Chebyshev error over simple family of functions). Local, global minima. Convex polyhedra (review matrix, scalar product geometry). Geometric picture of linear programming.

c. Lagrange multipliers and duality (6 lessons). Classical problem with equality constraints. Kuhn-Tucker conditions for inequality constraints. Linear programs. Dual variables as Lagrange multipliers. Reciprocity, duality theorems.

d. Solution of linear programs; simplex method (3 lessons).

e. Combinatorial problems (6 lessons). Unimodular property. Assignment problem (Hall theorem, unique representatives). Networks (min cut-max flow).

f. Classical calculus of variations (7 lessons). Stationarity. Euler's differential equation, gradient in function space. Examples, especially Fermat's principle and brachistochrones.

g. Control theory (8 lessons). Formulation. Pontryagin's maximum principle (Lagrange multipliers again).

PRINCIPAL REFERENCES

1. Hadley, G. LINEAR PROGRAMMING. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

2. Saaty, T. L., and Bram, J. NONLINEAR MATHEMATICS. New York: McGraw-Hill Book Co., 1964.

3. Weinstock, R. CALCULUS OF VARIATIONS. New York: McGraw-Hill Book Co., 1952.

OTHER REFERENCES

1. Athans, M., and Falb, P. OPTIMAL CONTROL. New York: McGraw-Hill Book Co., 1966.

2. Dantzig, G. B. LINEAR PROGRAMMING AND EXTENSIONS. Princeton, New Jersey: Princeton University Press, 1963.

3. Ford, L. R., Jr., and Fulkerson, D. R. FLOWS IN NETWORKS. Princeton, New Jersey: Princeton University Press, 1962.

4. Gale, D. THE THEORY OF LINEAR ECONOMIC MODELS. New York: McGraw-Hill Book Co., 1960.

5. Hadley, G. NONLINEAR AND DYNAMIC PROGRAMMING. Reading, Massachusetts: Addison-Wesley Publishing Co., 1964.

OPERATIONS RESEARCH IN SYSTEMS ENGINEERING

Year	Core Courses		Option Courses	
III	1. Functions of Several Variables	(3)		
	2. Intermediate Ordinary Differential Equations	(3)		
	3. Mechanics	(6)		
	4. Numerical Analysis	(3)		
	5. Probability and Statistics	(6)		
IV	6. Complex Variables	(3)	OR1. Reliability	(6)
	7. Functional Analysis	(3)		
	8. Electromagnetics	(3)		
	9. Thermodynamics and Statistical Mechanics	(3)		
	11. Optimization	(3)		
V.	10. Partial Differential Equations	(6)	OR2. Operations Research	(6)
			OR3. Systems Simulation	(3)

ORI. Quantitative Methods in Reliability Engineering (6 semester hours).

The growing complexity of systems over the last two decades has inspired the development of a body of quantitative methods for developing, improving, and measuring system reliability. Since reliability is such a critical factor in the successful completion of every highly technical program, it is of value to the engineer to learn quantitative reliability theory, as presented in the following course. The mathematical and statistical methods of the course are not simply routine applications of well-known theory, but in many cases represent new developments motivated by reliability problems.

The six semester hour course in Probability and Statistics of the core is an essential prerequisite for this course.

First Semester: Probabilistic Models in Reliability.

a. Failure distributions in reliability theory (8 lessons). Typical failure laws. The exponential as the failure law of complex equipment. Monotone failure rates. Bounds for distributions with monotone failure rate. General failure rates.

b. Prediction of system reliability from a knowledge of component reliabilities (3 lessons). Analytical methods for computing reliability exactly. Bounds on system reliability based on paths or cuts. Monte Carlo methods. Qualitative relationships for multicomponent structures.

c. Redundancy optimization (5 lessons). Optimal allocation of redundancy subject to constraints. Optimal redundancy assuming two kinds of failure.

d. Operating characteristics of maintenance policies (6 lessons). Renewal theory. Replacement based on age. Comparison of age and block replacement policies. Random replacement. Repair of a single unit.

e. Optimum maintenance policies (4 lessons). Replacement policies. Inspection policies.

f. Stochastic models for complex systems (8 lessons). Semi-Markov processes. Repairman problems. Marginal checking. Optimal maintenance policies under Markovian deterioration.

Second Semester: Statistical Reliability Theory.

a. Estimating reliability parameters assuming form of distribution known (8 lessons). Maximum likelihood estimation in the case of normal, exponential, gamma, Weibull, and binomial distributions. Confidence and tolerance limits in these cases. Minimum variance unbiased estimation in these cases.

b. Estimating reliability parameters under physically plausible assumptions (9 lessons). Errors resulting from incorrect assumption as to form of failure distribution. Maximum likelihood estimation assuming a monotone failure rate. Maximum likelihood estimation assuming a decreasing and then increasing failure rate. Conservative confidence and tolerance limits.

c. Estimating reliability growth (7 lessons). Form of growth assumed known. Only monotonicity of reliability assumed. Conservative confidence limits.

d. Confidence limits on system reliability using observations on individual components (6 lessons). Success or failure observations. Life length observations. Asymptotic methods.

e. Hypothesis testing (9 lessons). Acceptance sampling, fixed sample size, truncated and censored sampling, sequential sampling. Accelerated life testing. Testing for monotone failure rate.

PRINCIPAL REFERENCE

1. Barlow, R. E., and Proschan, F. THE MATHEMATICAL THEORY OF RELIABILITY. New York: John Wiley & Sons, 1965.

ADDITIONAL REFERENCES

1. Cox, D. R. RENEWAL THEORY. New York: John Wiley & Sons, 1962.

2. Cox, D. R. STATISTICAL THEORY OF RELIABILITY. (Edited by Marvin Zelen). Madison, Wisconsin: University of Wisconsin Press, 1963.

3. Cox, D. R., and Smith, W. L. QUEUES. New York: John Wiley & Sons, 1961.

4. Lloyd, D. K., and Lipow, M. RELIABILITY: MANAGEMENT, METHODS AND MATHEMATICS. Englewood Cliffs, New Jersey: Prentice-Hall, 1962.

OR2. Operations Research (6 semester hours).

The accent is on the mathematical aspects of the subject, rather than the management or industrial engineering aspects. It is assumed that time and facilities are available for a computation laboratory in connection with both semester courses. A prerequisite of an introductory computer programming course is desirable.

The first semester develops linear programming in depth, building on the preparation given in a previous one semester course in Optimization.

First Semester: Advanced Linear Programming.

a. Review of the simplex algorithm (4 lessons). Variations of the simplex algorithm. Degeneracy, perturbation. Revised simplex method.

b. Games and linear programs (4 lessons). Matrix games. Equivalence of matrix games and linear programs.

c. The transportation problems (4 lessons). Elementary transportation theory. The transshipment problem.

d. Networks and the transshipment problem (4 lessons). Graphs and trees. Interpreting the simplex method on the network. The shortest route problem.

e. Variables with upper bounds (3 lessons). The general case. The rounded variable transportation problem.

f. Programs with variable coefficients (3 lessons). Wolfe's generalized program. Special cases.

g. Decomposition principle for linear programs (9 lessons). The general principle. Decomposing multi-stage programs.

h. Convex programming (4 lessons). General theory. Separable convex objectives. Quadratic programming.

i. Discrete variable problems (4 lessons). Survey of methods. Gomory's method of integer forms.

Second Semester: Dynamic Programming and Stochastic Models.

a. Dynamic programming (10 lessons). Principle of optimality. Multi-stage allocation problems. Arrow-Harris-Marschak inventory model.

b. Dynamic programming and Markov processes (5 lessons). Discrete dynamic programming. Optimal policies with discounted returns.

c. Monte Carlo techniques (10 lessons). Production of random variables by computer. Simulating stochastic systems on the computer.

d. Mathematical theory of queues (14 lessons). Single server; Poisson input; exponential service. Many servers; Poisson input; exponential service. The busy period. Stochastic inventory models.

REFERENCES

Principal Reference.

1. Dantzig, G. B. LINEAR PROGRAMMING AND EXTENSIONS. (Chapters 11-26). Princeton, New Jersey: Princeton University Press, 1962.

Other Important References.

1. Bellman, R. DYNAMIC PROGRAMMING. Princeton, New Jersey: Princeton University Press, 1957.

2. Blackwell, D. "Discrete Dynamic Programming." ANNALS OF MATHEMATICAL STATISTICS 33 (1962), pp. 719-726.

3. Hadley, G. F. NONLINEAR AND DYNAMIC PROGRAMMING. Reading, Massachusetts: Addison-Wesley Publishing Co., 1964.

4. Marsaglia, G. RANDOM VARIABLES AND COMPUTERS. Boeing Scientific Research Laboratories Document DI-82-0182.

5. Prabhu, N. U. QUEUES AND INVENTORIES. New York: John Wiley & Sons, 1965.

Additional References.

1. Aris, R. DISCRETE DYNAMIC PROGRAMMING. New York: Blaisdell Publishing Co., 1964.

2. Cox, D. R., and Smith, W. L. QUEUES. New York: John Wiley & Sons, 1961.

3. Ford, L. R., Jr., and Fulkerson, D. R. FLOWS IN NETWORKS. Princeton, New Jersey: Princeton University Press, 1962.

4. Gale, D. THE THEORY OF LINEAR ECONOMIC MODELS. New York: McGraw-Hill Book Co., 1960.

5. Hadley, G. LINEAR PROGRAMMING. Reading, Massachusetts: Addison-Wesley Publishing Co., 1961.

6. Riordan, J. STOCHASTIC SERVICE SYSTEMS. New York: John Wiley & Sons, 1962.

7. Saaty, T. L., and Bram, J. NONLINEAR MATHEMATICS. New York: McGraw-Hill Book Co., 1964.

8. Shubik, M. GAME THEORY AND RELATED APPROACHES TO SOCIAL BEHAVIOR. New York: John Wiley & Sons, 1964.

9. Vajda, S. MATHEMATICAL PROGRAMMING. Reading, Massachusetts: Addison-Wesley Publishing Co., 1961.

10. Wilde, D. J. OPTIMUM-SEEKING METHODS. Englewood Cliffs, New Jersey: Prentice-Hall, 1963.

OR3. Systems Simulation (3 semester hours).

This course examines those symbol manipulation applications of the computer that involve the numerical and logical representation of some existing or proposed system, for the purpose of experimenting with the model and of comparing methods of operating the system. The primary purpose of the computer is thus not a calculating adjunct to experimentation but is the experimental medium itself. A course in probability and statistics is a prerequisite.

a. Programming languages (11 lessons). Special languages designed for use in simulation, such as SIMSCRIPT and GPSS. Additional study of the languages will arise in their use throughout the rest of the course.

b. Technical problems of simulation (14 lessons). Synchronization of events, file maintenance, random number generation, random deviate sampling.

c. Statistical problems peculiar to simulation (7 lessons). Sample size estimation, variance reducing techniques, problems of drawing inference from a continuous stochastic process.

d. Applications (7 lessons). Queueing models; storage, traffic, and feedback systems; design of facilities and operating disciplines.

REFERENCES

1. Markowitz, H. M., Hausner, B., and Karr, H. W. SIMSCRIPT: A SIMULATION PROGRAMMING LANGUAGE. Englewood Cliffs, New Jersey: Prentice-Hall, 1962.

2. Tocher, K. D. THE ART OF SIMULATION. New York: D. Van Nostrand Co., 1963.

ORBIT MECHANICS OPTION

Year	Core Courses		Option Courses	
III	1. Functions of Several Variables	(3)		
	2. Intermediate Ordinary Differential Equations	(3)		
	3. Mechanics	(6)		
	4. Numerical Analysis	(3)		
	5. Probability	(3)		
IV	6. Complex Variables	(3)	OM1. Advanced Numerical Analysis	(3)
	7. Functional Analysis	(3)		
	8. Electromagnetics	(3)		
	9. Thermodynamics and Statistical Mechanics	(3)		
	10. Partial Differential Equations	(6)		
V	11. Optimization	(3)	OM2. Advanced Programming	(3)
			OM3. Celestial Mechanics	(3)
			OM4. Orbit Theory	(3)
			CT2. Control	(3)
			CT5. Data Smoothing and Prediction	(3)

OM1. Advanced Numerical Analysis (3 semester hours).

This course, with its emphasis on topics in partial differential equations and elementary functional analysis, demands a reasonable amount of mathematical maturity. It should be taken after the first semester of Partial Differential Equations.

a. Matrix inversion and matrix eigenvalues (10 lessons). Review and extension of iterative methods. Jacobi, Householder and other methods of finding eigenvalues. Ill-conditioning and error analysis.

b. Ordinary differential equations, boundary value problems, eigenvalue problems (11 lessons). Finite difference methods, extremal principles.

c. Partial differential equations of second order (18 lessons). Topics selected from the following: Classification, analytical solutions of well-posed problems for single equations; maximum principles for elliptic and parabolic equations, L_2—or energy—estimates as well as pointwise estimates of solutions; hyperbolic equations, domain of dependence; Fourier analysis and stability for constant coefficient equations, eigenvalues for elliptic equations, iterative methods for difference equations arising from partial differential equations.

REFERENCES

1. Forsythe, G. E., and Wasow, W. R. FINITE DIFFERENCE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1960.

2. Fox, L. INTRODUCTION TO NUMERICAL LINEAR ALGEBRA, WITH EXERCISES. New York: Oxford University Press, 1965.

3. Hamming, R. W. NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS. New York: McGraw-Hill Book Co., 1962.

4. Henrici, P. DISCRETE VARIABLE METHODS IN ORDINARY DIFFERENTIAL EQUATIONS. New York: John Wiley & Sons, 1962.

5. Householder, A. S. THEORY OF MATRICES IN NUMERICAL ANALYSIS. New York: Blaisdell Publishing Co., 1964.

6. Ralston, A. FIRST COURSE IN NUMERICAL ANALYSIS. New York: McGraw-Hill Book Co., 1965.

7. Stiefel, E. L. AN INTRODUCTION TO NUMERICAL MATHEMATICS. New York: Academic Press, 1963.

8. Todd, J. SURVEY OF NUMERICAL MATHEMATICS. New York: McGraw-Hill Book Co., 1962.

9. Varga, R. S. MATRIX ITERATIVE ANALYSIS. Englewood Cliffs, New Jersey: Prentice-Hall, 1962.

10. Wendroff, B. THEORETICAL NUMERICAL ANALYSIS. New York: Academic Press, 1966.

11. Wilkinson, J. P. THE ALGEBRAIC EIGENVALUE PROBLEM. New York: Oxford University Press, 1965.

OM2. Advanced Programming (3 semester hours).

This course deals with various types of computer programming and serves to introduce students to the concepts involved in current work in this area. An introductory course in computer science is a prerequisite, and it is assumed that the students have considerable facility in programming with FORTRAN or ALGOL.

a. Survey (9 lessons). Assembly systems, methods of storage allocation when using these, pseudo-orders, macros, modify and load techniques, monitor and executive systems.

b. Structure of languages (15 lessons). Study of a particular language such as ALGOL, its ambiguities, its method of dealing with recursions and procedures. List processing languages, compiler writing languages.

c. Theory of compilers (15 lessons). Nature of syntax directed compilers, compilers for dealing with problem oriented languages, compilers for dealing with compiler syntax languages. Discussion of the evolution of a translator from a simple language whose translator is given in machine language.

REFERENCES

1. Bauer, F. L., Feliciano, M., Samelson, K., and Baumann, R. INTRODUCTION TO ALGOL. Englewood Cliffs, New Jersey: Prentice-Hall, 1964.

2. Iverson, K. E. PROGRAMMING LANGUAGE. New York: John Wiley & Sons, 1962.

3. Sherman, P. M. PROGRAMMING AND CODING DIGITAL COMPUTERS. New York: John Wiley & Sons, 1963.

OM3. Celestial Mechanics (3 semester hours).

This course in celestial mechanics concerns itself with the mathematical structures underlying the physical theory, and with deriving from them methods which are commonly used to attack the fundamental problems of interest to space research and technology.

a. Introduction (6 lessons). Description of dynamical systems by means of Lagrangian functions. Lagrangian equations of motion. Ignorable coordinates, energy equation and other elementary instances of first integrals. Liouville systems in general. Classical examples: harmonic oscillator, simple pendulum, spherical pendulum, central forces, a charged particle in an electromagnetic field, solid body. The principle of dynamical analogy. These lessons could be based on [9].

b. Phase space (6 lessons). Legendre duality (with a suggestion about how it is applied to derive state functions in classical thermodynamics). Transition from Lagrangian functions and Lagrangian equations to Hamiltonian functions and canonical equations. Canonical mapping: its definition, its multiplier and its residual functions. Completely canonical mappings; canonical extensions of coordinate transformations. Generation of canonical mappings by numerical functions. Invariance with respect to the group of canonical mappings: canonical equations, Poisson brackets, Lagrange parentheses. Examples and illustrations can be taken from [8] or [13]; for the theoretical exposition see [14].

c. Canonical constants of a dynamical system (6 lessons). Definition of a set of canonical constants of integration. Variation of canonical constants. The Hamilton-Jacobi equation as an algorithm for constructing sets of canonical constants. The action and angle variables. Separable Hamiltonians; Staeckel systems and Liouville systems. Applications: the problem of two bodies, the problem of two fixed centers. Normal modes of vibrations and vibrations of molecules. For the theoretical exposition, again refer to [5] and [14]. On Staeckel systems, either Staeckel's original papers or [3]. On vibrations of molecules in classical mechanics see [12].

d. Integrals of a dynamical system (9 lessons). Poisson's theorem about the bracket of two integrals and its dual application to Lagrange parentheses. Integrals in involution; Liouville's theorem. Jacobi-last multiplier. Application to the motion of a solid body. Whittaker's adelphic integral. Application to the investigation of a dynamical system around the equilibrium. Isoenergetic reduction. Application to the regularization and the binary collisions in the problem of two bodies and in the restricted problem of three bodies, refer to [13]. For the application of Jacobi's theory of the last multiplier to the motion of a solid body, refer to [7]. On the isoenergetic reduction and some of its applications, see [14].

e. Perturbation theory (12 lessons). Poincaré's method of the small parameter (for exposition, existence theorems, applications, see

[4] and [6]). Birkhoff's method of iterative canonical mappings (see [1], [11], and [12]). Application to the motion of a satellite of an oblate planet (see [2]).

REFERENCES

1. Born, M. MECHANICS OF THE ATOM. New York: Frederick Ungar Publishing Co., 1959.

2. Brouwer, D., and Clemence, G. M. METHODS OF CELESTIAL MECHANICS. New York: Academic Press, 1961.

3. Charlier, C. V. L. DIE MECHANIK DES HIMMELS. Leipzig: Veit, 1907.

4. Chazy, J. MÉCHANIQUE CÉLESTE. Paris: Presse Universitaire de France, 1953.

5. Goldstein, H. CLASSICAL MECHANICS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1950.

6. Golubew, W. W. (=V. V. Golubev). VORLESUNGEN ÜBER DIFFERENTIALGLEICHUNGEN IM KOMPLEXEN. Berlin: Deutscher Verlag der Wissenschaften, 1958.

7. Golubew, W. W. (=V. V. Golubev). LECTURES ON INTEGRATION OF THE EQUATIONS OF MOTION OF A RIGID BODY ABOUT A FIXED POINT. Washington, D. C.: Office of Technical Services, U. S. Department of Commerce, 1960.

8. Kilmister, C. W. HAMILTONIAN DYNAMICS. New York: John Wiley & Sons, 1964.

9. Landau, L. D., and Lifshitz, E. MECHANICS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1960.

10. Pollard, H. MATHEMATICAL INTRODUCTION TO CELESTIAL MECHANICS. Englewood Cliffs, New Jersey: Prentice-Hall, 1966.

11. Siegel, C. L. VORLESUNGEN ÜBER HIMMELSMECHANIK. Berlin: Springer-Verlag, 1956.

12. Ter Haar, D. ELEMENTS OF HAMILTONIAN MECHANICS. New York: John Wiley & Sons, 1964, second edition.

13. Whittaker, E. T. TREATISE ON THE ANALYTICAL DYNAMICS OF PARTICLES AND RIGID BODIES. New York: Cambridge University Press, 1959.

14. Wintner, A. ANALYTICAL FOUNDATIONS OF CELESTIAL MECHANICS. Princeton, New Jersey: Princeton University Press, 1941.

OM4. Orbit Theory (3 semester hours).

The purpose of this course is to offer illustrations of mathematical principles and to compel the student to master them securely by careful numerical examples. The material should be arranged to provide a nearly continuous flow of computational work for the laboratory sessions no matter at what level the course is set.

The instructor and his students should be directed to develop the topics all the way down to an efficient and reliable program in the FORTRAN or ALGOL language on an electronic computer. As most of the textbooks on the subject cater to computers who use logarithms or hand-operated desk model calculating machines, special care should be taken to rearrange classical algorithms and formulas for use on an electronic computer.

Topical problems should be selected from at least these four main research areas:

a. Orbit determination (10 lessons). The obvious reference here is [3]. This booklet has been updated by the author in his lectures on "Practical Astronomy" and on "Orbit Determination" given at the summer course in Space Mathematics, Cornell University, 1963.

b. Orbit analysis (12 lessons). The question here is to gain physical information from the comparison between the orbit as it has been observed and the orbit as it has been computed from a particular mathematical model. This problem is quite recent. Nothing definite has been published on it yet in the form of a textbook but the instructor will find it.

covered adequately in [1] and [6] .

c. Orbit design (7 lessons). How to produce orbits that satisfy *a priori* conditions (e.g., given initial conditions, mission requirements, optimum characteristics, etc.).

The course should limit itself to well tried problems and should aim at producing examples where good quality results can be reached without too much effort. This can be achieved in the restricted problem of three bodies.

After an introduction to that problem, the instructor should review two or three methods for integrating numerically the equations of motion, either in Cartesian coordinates or in regularized coordinates. Then should come a development on variational equations. Thereafter the theory of characteristic exponents (reference [7], pages 102-111) should be applied to the analysis of a family of periodic orbits.

d. Analytical theories (10 lessons). How to expand on literal theory by enabling an electronic computer to handle symbol manipulations in a given algebra.

This new field promises to provide mathematicians with powerful tools to develop literal theories in an extensive set of physical problems. References now available in this field are [2], [4], and [5]. Other publications are expected soon.

REFERENCES

1. Anderson, J. D. THEORY OF ORBIT DETERMINATION. Part I, Classical Methods; Part II, Estimation Formulas. Pasadena, California: Jet Propulsion Laboratory. TR-32-497 and TR-32-498. October 1, 1963.

2. Gerard, J. M., Iszak, I. G., and Barnett, M. P. "Mechanization of tedious algebra: The Newcomb operators of planetary theory." COMMUNICATIONS OF THE ASSOCIATION FOR COMPUTING MACHINERY 8(1965), pp. 27-32.

3. Herget, P. THE COMPUTATION OF ORBITS. Published privately by the author at Cincinnati Observatory, 1948.

4. Iszak, I. RESEARCH IN SPACE SCIENCE SPECIAL REPORTS NO. 129, 140, and 164. Smithstonian Astrophysical Observatory, 1963-64.

5. Kovalevsky, J. "Methode Numérique de Calcul de Perturbations Générales. Application au VIII^e Satellite de Jupiter." BULLETIN ASTRONOMIQUE 23 (1959), pp. 1-89.

6. Solloway, C. B. ELEMENTS OF THE THEORY OF ORBIT DETERMINATION. Pasadena, California: Jet Propulsion Laboratory. EPD-255. December 9, 1964.

7. Szebehely, V. G. PERIODIC ORBITS IN THE RESTRICTED PROBLEM OF THREE BODIES. New York: Academic Press, 1966.

8. Wintner, A. ANALYTICAL FOUNDATIONS OF CELESTIAL MECHANICS. Princeton, New Jersey: Princeton University Press, 1941.

CONTROL THEORY OPTION

Year	Core Courses		Option Courses	
III	1. Functions of Several Variables	(3)	CT1. Circuits	(3)
	2. Intermediate Ordinary Differential Equations	(3)		
	3. Mechanics	(6)		
	4. Numerical Analysis	(3)		
	5. Probability and Statistics	(6)		
IV	6. Complex Variables	(3)	CT2. Control	(3)
	7. Functional Analysis	(3)	CT3. Laboratory	(3)
	8. Electromagnetics	(3)	CT4. Linear Systems	(3)
	9. Thermodynamics and Statistical Mechanics	(3)	CT5. Data Smoothing and Prediction	(3)
V	10. Partial Differential Equations	(6)	CT6. Advanced Control	(3)
			CT7. Information Theory	(3)
	11. Optimization	(3)	CT8. Advanced Communications	(3)

CT1. Electric Circuits (3 semester hours).

This is a basic course in circuit theory. The purpose is to teach in a precise language the fundamental facts of circuit theory while developing skills in writing and solving the circuit equations and keeping close contact with physical circuits (filters, amplifiers, digital circuits).

a. Lumped circuits (3 lessons). Lumped circuit approximation. Kirchhoff laws, relation to Maxwell's equations. Circuit elements; including nonlinear and time-varying elements.

b. Simple circuits (13 lessons). First and second order circuits: zero-input response and zero-state response; response to step, impulse and sinusoid. Linearity and time-invariance: convolution. Impedance, phasors, frequency response and resonance.

c. Coupling elements (2 lessons). Coupled inductors, transformers, and dependent sources. Dependent sources as parts of models for electronic devices.

d. Power and energy (2 lessons). Energy stored and power dissipated in elements; relation with real and imaginary part of impedance.

e. General methods of analysis (6 lessons). Graph theory: trees, links, cut-sets, loops. Loop and cut-set analysis, mixed method. Duality. Computer programs for analysis of circuits.

f. Linear time-invariant circuits (6 lessons). Reduction of systems of equations. Network functions: poles, zeros, gain and phase.

g. Network theorems (3 lessons). Superposition, Thévenin, Norton,

reciprocity. Careful discussion of range of applicability: comments on nonlinear and time-varying circuits.

h. Two-port description (4 lessons). Two-port description of electronic devices: relation to their graphical characteristics. Linear time-invariant networks as two-ports. Interconnection of two-ports.

REFERENCES

1. Bose, A. G., and Stevens, K. INTRODUCTORY NETWORK THEORY. New York: Harper & Row, Publishers, 1965.

2. Desoer, C. A., and Kuh, E. S. BASIC CIRCUIT THEORY. New York: McGraw-Hill Book Co., 1966.

3. Friedland, B., Wing, O., and Asch, R. B. PRINCIPLES OF LINEAR NETWORKS. New York: McGraw-Hill Book Co., 1961.

CT2. Control (3 semester hours).

In this course the student learns some basic facts about control systems, their analytical description, and techniques of design. This course is mostly concerned with single variable control.

a. Description of feedback systems and components (5 lessons). Advantages and disadvantages of feedback, importance of measuring device, noise problems. Basic components, electrical, hydraulic, pneumatic. Requirements and specifications of control systems. Examples.

b. Linear time-invariant control systems (20 lessons). Analysis, illustrated by several extensive examples, based on differential equations and integral equations (convolution). Stability, root locus, Nyquist criterion. Design of compensating networks to obtain stability and meet the specifications.

c. Sampled systems (6 lessons). Examples of systems where the feedback data is naturally sampled periodically. Analysis of sampled systems. Stability, root locus, Jury's criterion. Design of compensating networks.

d. Nonlinear systems (8 lessons). Local stability near equilibrium. Examples of limit cycles. Stability: approximate methods describing functions. Lyapunov's second method. Application to design.

REFERENCES

1. Chestnut, H., and Mayer, R. W. SERVOMECHANISMS AND REGULATING SYSTEM DESIGN. New York: John Wiley & Sons, second edition, Volume I, 1959; Volume II, 1955.

2. D'Azzo, J. T., and Houpis, C. H. CONTROL SYSTEM ANALYSIS AND SYNTHESIS. New York: McGraw-Hill Book Co., 1966, second edition.

3. Gille, J. C., Pelegrin, M. J., and DeCaulne, P. FEEDBACK CONTROL SYSTEMS. New York: McGraw-Hill Book Co., 1959.

4. Horowitz, I. M. SYNTHESIS OF FEEDBACK SYSTEMS. New York: Academic Press, 1963.

5. James, H. M., Nichols, N. B., and Phillips, R. S. THEORY OF SERVOMECHANISMS. M.I.T. Radiation Laboratory Series No. 25. New York: McGraw-Hill Book Co., 1947.

6. Jury, E. I. THEORY AND APPLICATIONS OF THE Z-TRANSFORMATION METHOD. New York: John Wiley & Sons, 1964.

7. Newton, G. C., Gould, L. A., and Kaiser, J. F. ANALYTICAL DESIGN OF LINEAR FEEDBACK CONTROLS. New York: John Wiley & Sons, 1957.

8. Truxal, J. G. AUTOMATIC FEEDBACK CONTROL SYSTEM SYNTHESIS. New York: McGraw-Hill Book Co., 1955.

9. Tsien, H. S. ENGINEERING CYBERNETICS. New York: McGraw-Hill Book Co., 1954.

CT3. Laboratory (3 semester hours).

The purpose of the laboratory is to insure that students connect the classroom concepts and results to physical reality and appreciate the power and limitations of experimental work. Typically one part of the laboratory could be devoted to circuits work: behavior of linear circuits (including resonance), effects of nonlinear elements on waveform and power spectrum, some pulse circuits. The second part of the laboratory would cover control: study of a typical control system, experiments with various compensations; stability; experiments with and simulation of a nonlinear control system.

CT4. Linear Systems (3 semester hours).

Purpose: to provide a solid foundation of concepts, facts and techniques to be used in later courses in control, communication, and circuits.

a. Systems (6 lessons). State as a parametrization of input-output pairs and as part of the system description. Operator point of view. State equivalence. Linear systems. Linear systems obtained by linearization of ordinary differential equation about a nominal trajectory. Examples throughout.

b. Linear systems of the form $\begin{cases} \dot{x} = Ax + By \\ \dot{y} = Cx + Dy \end{cases}$ and their discrete time analogs (6 lessons). Time-invariant case: explicit solution by function of a matrix and Laplace and z-transforms. For simple linear operators, diagonalization, mode interpretation (including numerical techniques). Jordan form, analog computer interpretation. Time-varying case: properties of the state transition matrix. Periodic systems: Floquet theory, kinematic equivalence.

c. Impulse response and transfer functions (12 lessons). Free use of Fourier and Laplace transforms. Superposition integral. Asymptotics of impulse response and transfer function. Minimum phase. Uncertainty principle. Group delay. Signal flow graphs.

d. Stability (9 lessons). Characterization of stability for linear time invariant, periodic and time varying systems (zero-input stability: Liénard-Chipart, Nyquist; bounded input implies bounded output; implications of an impulse response which is in L^1), Lyapunov method.

e. Input-output description and state equations (6 lessons). Controllability, observability, and normality. Characterization in time-invariant and time-varying cases. Output controllability. Controllability and observability of an interconnection of systems.

REFERENCES

1. Athans, M., and Falb, P. OPTIMAL CONTROL. New York: McGraw-Hill Book Co., 1966.

2. DeRusso, P. M., Roy, R. J., and Close, C. M. STATE VARIABLES FOR ENGINEERS. New York: John Wiley & Sons, 1965.

3. Doetsch, G. HANDBUCH DER LAPLACE TRANSFORMATION. Basel: Birkhauser, Volume I, 1950; Volume II, 1955; Volume III, 1957.

4. Guillemin, E. A. THEORY OF LINEAR PHYSICAL SYSTEMS. New York: John Wiley & Sons, 1963.

5. Hochstadt, H. DIFFERENTIAL EQUATIONS. New York: Holt, Rinehart and Winston, 1964.

6. Kalman, R. E. "Mathematical Description of Linear Dynamical Systems." JOURNAL OF THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS. Series A: Control 1(1963), 152-192.

7. Kaplan, W. OPERATIONAL METHODS FOR LINEAR SYSTEMS. Reading, Massachusetts: Addison-Wesley Publishing Co., 1962.

8. Papoulis, A. FOURIER INTEGRAL AND ITS APPLICATIONS. New York: McGraw-Hill Book Co., 1962.

9. Schwarz, R., and Friedland, B. LINEAR SYSTEMS. New York: McGraw-Hill Book Co., 1965.

10. Zadeh, L. A., and Desoer, C. A. LINEAR SYSTEM THEORY. New York: McGraw-Hill Book Co., 1963.

CT5. <u>Data Smoothing and Prediction</u> (3 semester hours).

a. Representation of functions by Fourier series and integrals. The Fourier transform in L^1 and L^2. (8 lessons)

b. Random processes: definition, examples, representations; autocorrelation, power spectrum; estimation of spectral densities. (14 lessons)

c. Linear mean-square estimation, filtering and prediction. The Wiener-Hopf equation; solution by the Wiener filter and Kalman-Bucy filter. (11 lessons)

d. Detection and parameter estimation. Application to digital communications system and radar. (6 lessons)

REFERENCES

1. Blackman, R. B., and Tukey, J. W. MEASUREMENT OF POWER SPECTRA FROM THE POINT OF VIEW OF COMMUNICATIONS ENGINEERING. New York: Dover Publications, 1959.

2. Davenport, W. B., Jr., and Root, W. L. INTRODUCTION TO RANDOM SIGNALS AND NOISE. New York: McGraw-Hill Book Co., 1958.

3. Helstrom, C. W. STATISTICAL THEORY OF SIGNAL DETECTION. New York: Pergamon Press, 1960.

4. Middleton, D. INTRODUCTION TO STATISTICAL COMMUNICATION THEORY. New York: McGraw-Hill Book Co., 1960.

5. Papoulis, A. FOURIER INTEGRAL AND ITS APPLICATIONS. New York: McGraw-Hill Book Co., 1962.

6. Papoulis, A. PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES. New York: McGraw-Hill Book Co., 1965.

7. Parzen, E. STOCHASTIC PROCESSES. San Francisco: Holden-Day, 1962.

8. Woodward, P. M. PROBABILITY AND INFORMATION THEORY, WITH APPLICATIONS TO RADAR. New York: Pergamon Press, 1953.

CT6. Advanced Control (3 semester hours).

This course treats advanced topics in control so that students can readily read the current literature. Multiple-input multiple-output systems are included.

a. Nonlinear control (10 lessons). Describing function. Subharmonics. Stability: Sandberg circle criterion, Popov-type criteria. Lyapunov method used as a design tool. Lyapunov method for systems with inputs. Bounds on output.

b. Adaptive control (8 lessons). Examples of adaptive control: identification techniques and parameter adjustment. Stochastic approximation.

c. Optimum control (21 lessons). Formulation of the problem. Examples: systems described by ordinary differential equations and difference equations. Maximum principle for differential systems. Numerical methods. Relation of maximum principle with steepest descent.

REFERENCES

1. Athans, M., and Falb, P. OPTIMAL CONTROL. New York: McGraw-Hill Book Co., 1966.

2. Gibson, J. E. NONLINEAR AUTOMATIC CONTROL. New York: McGraw-Hill Book Co., 1963.

3. LaSalle, J., and Lefschetz, S. STABILITY BY LYAPUNOV'S DIRECT METHOD WITH APPLICATIONS. New York: Academic Press, 1961.

4. Minorsky, N. NONLINEAR OSCILLATIONS. Princeton, New Jersey: D. Van Nostrand Co., 1962.

5. Popov, E. P. THE DYNAMICS OF AUTOMATIC CONTROL SYSTEMS. New York: Pergamon Press, 1962.

6. Tou, J. T. MODERN CONTROL THEORY. New York: McGraw-Hill Book Co., 1964.

CT7. Information Theory (3 semester hours).

a. The concept of the source and of an information measure. Desirable properties of information measure, examples of simple sources. (3 lessons).

b. Codes, their efficiency and redundancy. The efficient encoding of discrete independent sources. (4 lessons).

c. General discrete sources, Shannon's encoding theorem, the nature of written and spoken English. (4 lessons).

d. The concept of a channel, channel capacity, symmetry of a channel. (3 lessons).

e. The fundamental theorem of information theory, error-detecting and error-correcting codes, the geometric interpretation of coding problems. (17 lessons).

f. Generalization to continuous channels, channel capacity of continuous channels. (8 lessons).

REFERENCES

1. Abramson, N. INFORMATION THEORY AND CODING. New York: McGraw-Hill Book Co., 1963.

2. Ash, R. B. INFORMATION THEORY. New York: John Wiley & Sons, 1965.

3. Fano, R. M. TRANSMISSION OF INFORMATION. Cambridge, Massachusetts: The M. I. T. Press, 1961.

CT8. Advanced Communications (3 semester hours).

a. Review (4 lessons). Signal and noise representations, the purpose of modulation.

b. Amplitude modulation (7 lessons). The generation and detection of AM waves, power spectrum, single side-band and vestigial side-band transmission, effects of distortion and noise.

c. Frequency and angle modulation (12 lessons). Generation, detection, power spectrum, effects of distortion and noise.

d. Pulse modulation (10 lessons). Pulse amplitude modulation, pulse position modulation, pulse duration modulation. A brief introduction to pulse code modulation.

e. Design (6 lessons). The design of optimum receivers in the presence of additive noise and fading.

REFERENCES

1. Helstrom, C. W. STATISTICAL THEORY OF SIGNAL DETECTION. New York: Pergamon Press, 1960.

2. Middleton, D. AN INTRODUCTION TO STATISTICAL COMMUNICATION THEORY. New York: McGraw-Hill Book Co., 1960.

3. Rowe, H. E. SIGNALS AND NOISE IN COMMUNICATION SYSTEMS. Princeton, New Jersey: D. Van Nostrand Co., 1965.

4. Wainstein, L. A., and Zubakov, V. D. EXTRACTION OF SIGNALS FROM NOISE. Englewood Cliffs, New Jersey: Prentice-Hall, 1962.

5. Wozencraft, J. W., and Jacobs, I. M. PRINCIPLES OF COMMUNICATION ENGINEERING. New York: John Wiley & Sons, 1965.

CONTENTS

COURSE OUTLINES